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Electrical Modeling of Piezoelectric Ceramics for Analysis and Evaluation of Sensory Systems

Jina Kim, Benjamin L. Grisso, Jeong K. Kim, Dong Sam Ha, and Daniel J. Inman

***Abstract*—Piezoelectricity is an ability of some materials to generate an electric potential in response to applied mechanical stress. Piezoelectric ceramics are often used for sensory systems to monitor mechanical characteristics of structures through an electrical signal. Thus, to support system level analysis and evaluation of sensory systems, understanding and estimating the electrical behavior of piezoelectric ceramics with a minimum effort is valuable. This paper proposes an equivalent circuit model for piezoelectric ceramics, with a PZT patch as the specimen. The proposed model approximates the electrical behavior of piezoelectric ceramics with up to 100% accuracy for unloaded and 93% accuracy for loaded PZT, and provides a modeling procedure that can easily be automated.**

***Index Terms*—Equivalent circuits, Modeling, Piezoelectric materials, PZT ceramics**

I. INTRODUCTION

Piezoelectricity is an ability to generate an electric potential in response to applied mechanical stress [1]. As the piezoelectric effect is reversible, materials that demonstrate the direct piezoelectric effect, which is the generation of electricity upon applied mechanical stress, also demonstrate the converse piezoelectric effect, which is the generation of stress and strain upon applied electric field [2]. There are different types of piezoelectric materials, and some representative materials include quartz and Lead-Zirconate Titanate (PZT), which are a natural crystal and a man-made ceramic, respectively.

Impedance-based structural health monitoring (SHM) is one of the applications, where a piezoelectric ceramic is used. A piezoelectric transducer made of a single layer piezoelectric ceramic is usually utilized for impedance-based SHM as a collocated sensor/actuator to monitor mechanical conditions of a structure through electrical signals. PZT, which is a surface-mountable sensor/actuator in a form of a thin patch, is the most widely used piezoelectric ceramic due to its high actuation ability [1][3]. When an electric field is placed across the thickness of a PZT patch as the original polarization field, the PZT patch expands along the axis of polarization (thickness direction) and contracts perpendicular to the axis of polarization (length and width direction) [6]. PZT typically changes its shape up to 0.1% of the original dimension [1][2].

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For sensory systems relying on piezoelectricity, such as impedance-based SHM systems, it is important to model the piezoelectric behavior of a piezoelectric ceramic itself and the interaction between a piezoelectric ceramic and the structure in terms of an electrical equivalent circuit, to conduct system level analysis and performance evaluation intensively. There are several previous approaches on modeling the piezoelectric behavior with an electrical equivalent circuit, but they do not provide a modeling procedure that can be easily automated, and do not present quantitative performance of their equivalent circuit models [3]-[5]. This work proposes a systematic procedure to model the piezoelectric behavior of an unloaded or loaded piezoelectric ceramic. It also presents quantitative performance of the proposed model in terms of correlation coefficients.

II. EXISTING EQUIVALENT CIRCUIT MODELS

A. Unloaded Piezoelectric Ceramics

The most basic equivalent circuit model characterizing a piezoelectric ceramic near the resonant frequency is the Van Dyke Model shown in Figure 1, which is often adopted to model electromechanical resonance characteristics of crystal oscillators [7][8]. The Van Dyke Model is a parallel connection of a series RLC representing mechanical damping, mass, and elastic compliance, and a capacitor representing the electrostatic capacitance between the two parallel plates of the PZT patch [7].

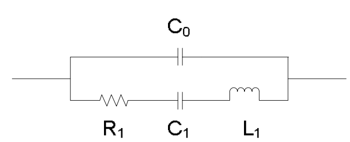


Figure 1 The Van Dyke Model

The Van Dyke Model cannot accurately model the material characteristics, particularly for materials with significant losses [4]. The Sherrit Model employs complex circuit components, as shown in Figure 2, to model the piezoelectric ceramic with piezoelectric losses. Circuit component values of the Sherrit Model are complex numbers, while those of the Van Dyke Model are real numbers.

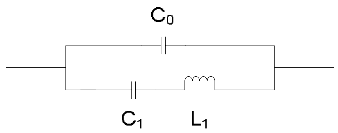


Figure 2 The Sherrit Model – Unloaded piezoelectric ceramics

The Guan Model, which is the most recently proposed, estimates values of the electrical components based on the electrical behavior of the piezoelectric ceramics [3]. The Guan model, as shown in Figure 3, adds a series resistor R_s and a parallel resistor R_p to C_0 of the Van Dyke Model to account for the energy dissipation. Determination of the values of the electric components C_0, C_1, L_1 and R_1 relies on visual inspection on the magnitude and phase of the impedance, and values of R_s and R_p are decided by the amount of energy dissipation [3].

The Guan Model attempts to improve the accuracy of the Van Dyke Model by encompassing R_s and R_p as a consideration of the energy dissipation. However, the values of R_s and R_p may introduce inaccuracy as the energy loss is dependent on the amplitude and the frequency of the excitation signal [3].

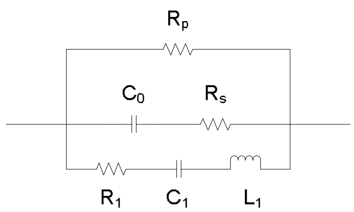


Figure 3 The Guan Model – Unloaded piezoelectric ceramics

B. *Loaded Piezoelectric Ceramics*

When a piezoelectric ceramic is mounted to a mechanical structure, the mechanical boundary conditions of the piezoelectric ceramic change [3], and, accordingly, a different circuit model is required for a loaded piezoelectric ceramic. Since a loaded piezoelectric ceramic experiences multiple resonances, a circuit model for a wide frequency range with multiple resonant frequencies can be employed to model the behavior of a loaded piezoelectric ceramic. As shown in Figure 4, additional R-C-L branches are added in parallel to the R_1-C_1-L_1 branch of the Van Dyke Model [7].

The Guan Model is also extended to accommodate a loaded piezoelectric ceramic based on the extended Van Dyke Model, as shown in Figure 5. Each series RLC branch physically stands for a mechanical resonant mode. The values of R_i, C_i, and L_i, for i from 1 to n, are determined by the same method to obtain the values of R_1, C_1, and L_1 discussed in Section II.A. The complete Guan Model has difficulty in determining values of R_i, C_i, and L_i when the resonant frequencies of the piezoelectric ceramic and structure are close to or overlapping with each other [3].

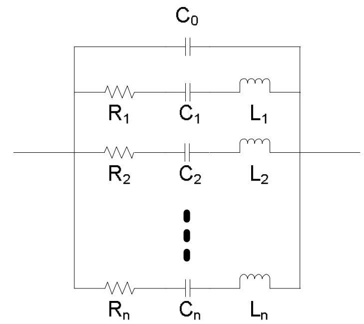


Figure 4 The extended Van Dyke Model – Loaded piezoelectric ceramics

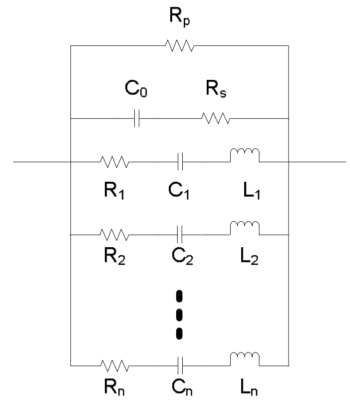
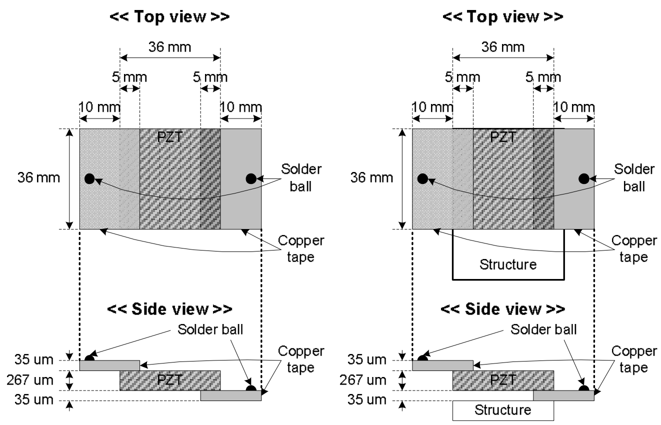


Figure 5 The complete Guan Model – Loaded piezoelectric ceramics

III. TEST CONFIGURATIONS

We present a systematic procedure to model a piezoelectric ceramic, and its model is called the *Easy Model*. As the first step to develop the Easy Model, the electrical properties of the piezoelectric ceramic are investigated by measuring the electrical impedance of the piezoelectric ceramic using an impedance analyzer such as HP 4194A. The impedance is measured from 10 KHz to 100 KHz with a frequency resolution of 20 Hz. A PZT sheet *PSI-5H4E-T110* available from Piezo Systems, Inc. is selected as a piezoelectric ceramic in developing the Easy Model and cut into a smaller piece to be used as a specimen.

The test configurations for unloaded PZT and loaded PZT are illustrated in Figure 6. The PZT patch is excited with an electrical signal to produce a mechanical actuation on the structure, and the mechanical response from the structure is measured through the PZT patch in the form of an electrical signal. Thus, a pair of wires should be connected to each face of the PZT patch. Since the PZT patch is attached to the surface of a structure, one side of a loaded PZT patch is not accessible. Thus, a piece of copper tape is used to provide a route to the bottom of the PZT patch that is contacting the structure as shown in Figure 6 (b). For the purpose of symmetry, a piece of copper tape with the same size is also attached to the top of the PZT patch, and q wire is soldered to each of the two pieces of copper tape. Since the copper tape thickness shown in Figure 6 is only 35 μm, a loaded PZT patch has almost full contact with the structure, and the gap between the PZT patch and a structure shown in Figure 6 (b) is not to scale. As presented in Figure 6 (a), unloaded PZT includes the PZT patch and the copper tape pieces to keep test conditions of the specimen identical for unloaded and loaded cases.



(a) Unloaded PZT (b) Loaded PZT

Figure 6 Test configurations (not to scale) – PZT

In the loaded PZT measurement and modeling cases, an aluminum beam is used as a test structure whose dimension and PZT bonding location is depicted in Figure 7.

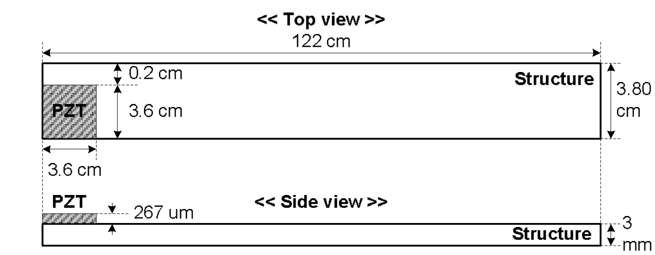


Figure 7 PZT on a test structure (not to scale)

IV. Proposed Easy Model for Unloaded Piezoelectric Ceramics

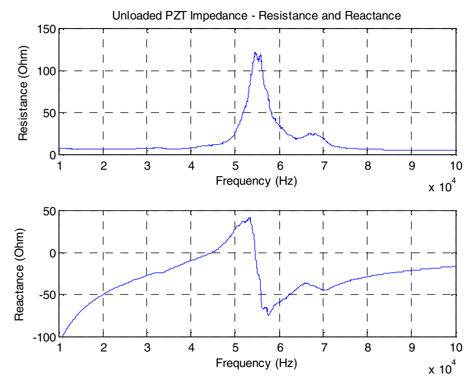
A. Proposed Model

The measured impedance of unloaded PZT is presented in Figure 8. From the reactance curve shown in Figure 8 (a) and the magnitude curve in Figure 8 (b), one can observe that the specimen has a series resonance at 45 KHz and a parallel resonance at 55 KHz. Note that the magnitude becomes the maximum (minimum) at the parallel (series) resonance. Figure 8 (a) indicates that the PZT patch is inductive between the series resonant frequency and the parallel resonant frequency and capacitive in other frequency ranges. In a low frequency range, the reactance approaches negative infinity as the frequency decreases. It can also be noted from the resistance curve that the base resistance outside the resonant frequency range is about 5 Ω.

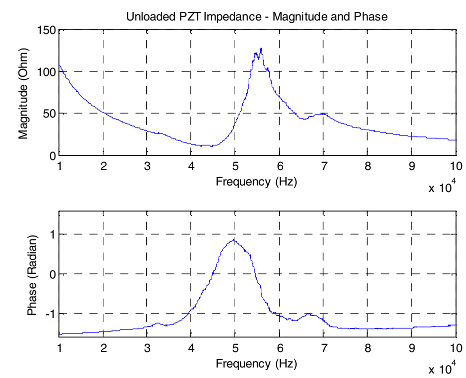
Since it is easy to identify the parallel resonant frequency from the measured impedance, the resonance characteristics of unloaded PZT can be modeled using a parallel RLC tank circuit. A resistor in series to the RLC tank circuit is connected, as unloaded PZT demonstrates nearly constant resistance over the frequency range away from the resonant frequency. Also, as the reactance converges to negative infinity as the frequency approaches DC, a series capacitor to an RLC tank circuit is added. The Easy Model is shown in Figure 9. The model has an RLC tank circuit with a resistor and a capacitor in series. The impedance of unloaded PZT through the Easy Model is given as:

$$Z_{pm,unloaded}(\omega) = R_0 + \frac{1}{j\omega C_0} + \frac{1}{\dfrac{1}{R_1} + \dfrac{1}{j\omega L_1} + j\omega C_1} \qquad (1)$$

Note that the Easy Model is another form of the Van Dyke Model. The series RLC branch and a parallel capacitor of the Van Dyke Model are transformed into a parallel RLC tank and a series capacitor in the Easy Model. The series to parallel conversion of the RLC components makes the calculation of the electrical component values of the model simple. So the Easy Model is suitable for automating the modeling process.



(a) Resistance and Reactance



(b) Magnitude and Phase

Figure 8 Unloaded PZT impedance – Measured

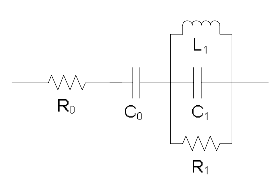


Figure 9 The Easy Model – Unloaded PZT

B. Determination of Component Values

The next step for the Easy Model is to determine the values of electrical components. The resistance value R_0 can be determined from the resistance curve at a DC or very high frequency. It is approximately 5 Ω from Figure 8 (a). The series capacitor value C_0 is from the reactance curve by taking a negative inverse of the reactance value at DC. It is 150 nF from Figure 8 (a). The parallel resonant frequency ω_p is determined also from the reactance curve, and is 55 KHz from Figure 8 (a). The resistance value of R_1 can be calculated as

$$R_1 = R_{p,unloaded}\left(\omega_p\right) - R_0 \tag{2}$$

where $R_{p,unloaded}\left(\omega_p\right)$ is the measured resistance of unloaded PZT at the parallel resonant frequency ω_p. The measured value of $R_{p,unloaded}\left(\omega_p\right)$ is 120 Ω for the above PZT, and hence R_1 is obtained as 115 Ω. Since $\omega_p = 1/\sqrt{LC}$ and the quality factor $Q = R\sqrt{C/L}$ [9], the value of L_1 can be calculated as

$$L_1 = \frac{R_1}{\omega_p \cdot Q} \tag{3}$$

and the value of C_1 can be obtained as

$$C_1 = \frac{1}{L_1 \cdot \omega_p^{\,2}} \tag{4}$$

The Q factor appeared in equation (3) can be obtained from the magnitude curve of Figure 8 (b) using a relationship $Q = \omega_p / BW$, where BW is the 3-dB bandwidth. The Q factor is obtained as 1100, and L_1 and C_1 as 30.253 µH and 0.277 µF, respectively, for the above PZT. Note that the above procedure is straightforward from the measured impedance and can easily be automated. The component values are summarized in Table 1.

TABLE 1 THE EASY MODEL CIRCUIT COMPONENTS – UNLOADED PZT

R_0 (Ω)	5	R_1 (Ω)	115
C_0 (µF)	0.15	C_1 (µF)	0.277
		L_1 (µH)	30.253

C. Performance Analysis

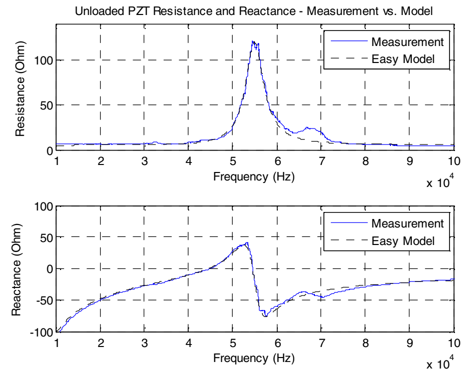
The dotted lines in Figure 10 show the estimated impedance of the specimen based on the Easy Model. From a visual inspection, the Easy Model follows the trends of the measured impedance in four aspects, resistance, reactance, magnitude and phase. Correlation coefficient indicates the strength and direction of a linear relationship between two variables [10]. Thus, for a quantitative comparison, correlation coefficients between the measured and the modeled impedance are calculated, and summarized in

Table 2. As seen in the table, the correlation coefficients are about 0.99 for all the four parameters.

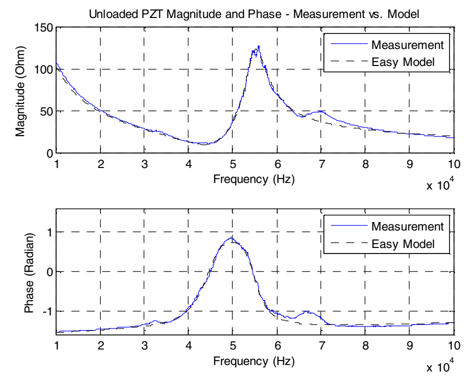
TABLE 2 UNLOADED PZT CORRELATION COEFFICIENTS – MEASUREMENT VS. EASY MODEL

Impedance Component	Correlation Coefficient
Resistance	0.9887
Reactance	0.9922
Magnitude	0.9932
Phase	0.9926

The unloaded PZT illustrated in this section includes impurities, two pieces of copper tape, as shown in Figure 6 (a), and the impurities cause a local resonant frequency around 68 KHz. The local resonant frequency is ignored for the unloaded PZT, but is discussed in the next section.



(a) Resistance and Reactance



(b) Magnitude and Phase

Figure 10 Unloaded PZT impedance – Measurement vs. Easy Model

V. EASY MODEL FOR LOADED PIEZOELECTRIC CERAMICS

A. Proposed Model

As mentioned in Section II.B, mechanical structures usually have multiple resonant frequencies depending on their mechanical characteristics. Thus, when a PZT patch is bonded to a structure, the resonant mode of unloaded PZT has interaction with each resonant mode of the structure, as shown in Figure 11. In the figure, n is the number of resonant frequencies of the structure, and the subscripts *pmu* and *s* denote unloaded PZT Model and Structure, respectively.

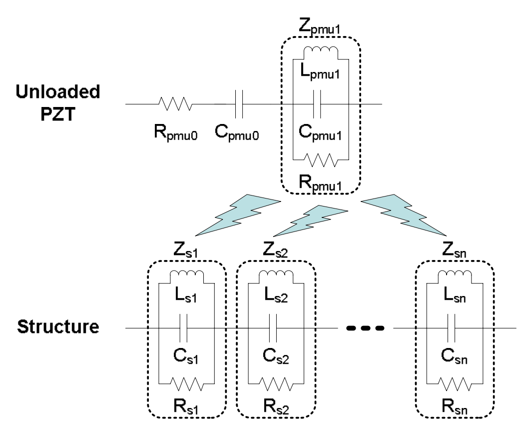


Figure 11 Interaction between unloaded PZT and a structure

The interaction between PZT and the structure can be explained with a concept of a transformer as shown in Figure 12. The top two figures of Figure 12 show the equivalent circuit model of PZT and a structure at a resonant frequency using an RLC tank circuit based on the Easy Model. The Z_{pmu1} block represents the resonance of PZT, and the Z_{si} block represents one of the resonances of the structure. When a PZT patch is bonded to the surface of a structure, the interaction between a PZT patch and a resonance frequency of the structure may be modeled as a transformer with a transfer ratio of N_p:N_s as shown in the second figure of Figure 12. Note the inductors of RLC tank circuits, L_{pmu1} and L_{si}, are distributed into two elements satisfying $L_{pmu1} = L_{pmu11} \parallel L_{pmu12}$ and $L_{si} = L_{si1} \parallel L_{si2}$.

As illustrated in the third figure of Figure 12, the inductance of PZT constituting the transformer, L_{pmu12}, can be replaced with the structure's impedance seen by PZT, Z_{si}', where $Z_{si}' = Z_{si} \cdot \left(N_p / N_s\right)^2$. The impedance of loaded PZT at a resonant frequency ω_i, Z_{pmli}, becomes a parallel connection of an RLC tank circuit of unloaded PZT excluding the inductance forming a transformer, Z_{pmu11}, and an impedance of the structure seen by PZT, Z_{si}', as expressed in equation (5).

$$Z_{pmli} = Z_{pmu11} \parallel Z_{si}' = R_{pmu1} \parallel C_{pmu1} \parallel L_{pmu11} \parallel Z_{si}' \tag{5}$$

Finally, by distributing RLC components of Z_{si}' into Z_{pmu11}, a new equivalent RLC tank circuit representing the behavior at a resonant frequency ω_i is obtained with R_{pmli}, C_{pmli}, and L_{pmli} as shown in the last row of Figure 12.

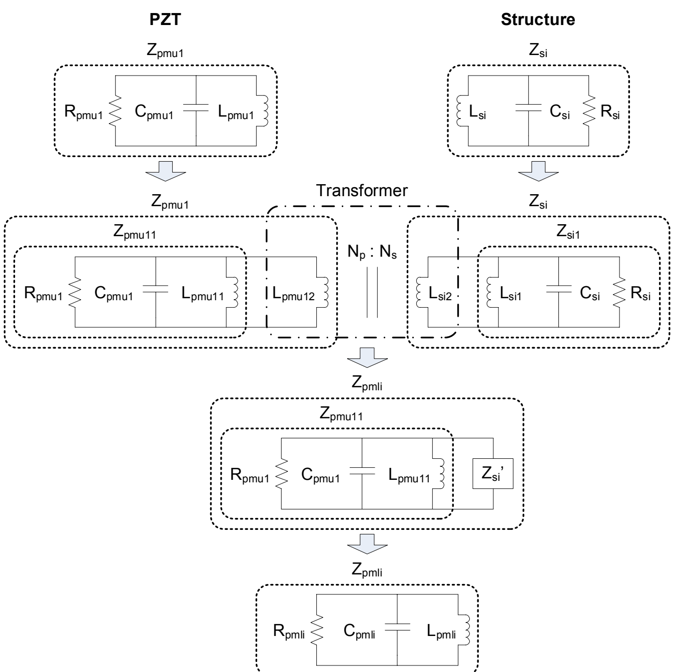


Figure 12 PZT and structure as a transformer

The procedure elaborated in Figure 12 is applied to all resonant frequencies of the structure, and the Easy Model for unloaded PZT is extended as shown in Figure 13 to compose the Easy Model for loaded PZT. Note the values of R_0 and C_0 should be determined according to the measured impedance of loaded PZT, rather than duplicating the values determined for an unloaded PZT model. The total estimated impedance of loaded PZT $Z_{pm,loaded}(\omega)$ becomes

$$Z_{pm,loaded}(\omega) = R_0 + \frac{1}{j\omega C_0} + \sum_{i=1}^{n} \frac{1}{\frac{1}{R_i} + \frac{1}{j\omega L_i} + j\omega C_i} \tag{6}$$

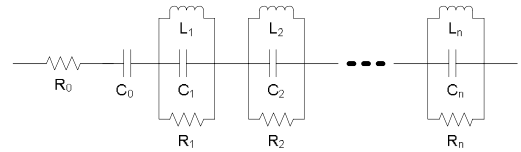
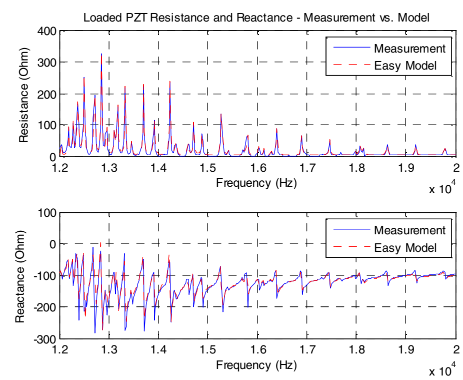


Figure 13 Easy Model – Loaded PZT

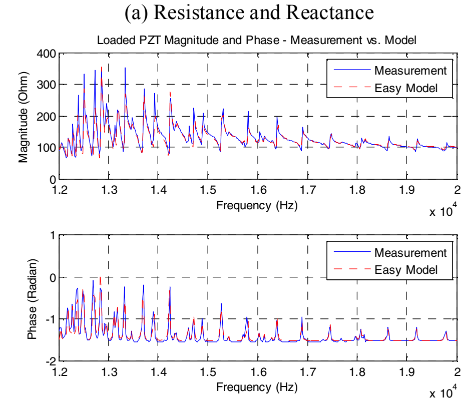
B. Performance Analysis

It is observed from the measured impedance curves that loaded PZT exhibits multiple resonant frequencies, especially in frequency ranges from 12 KHz to 20 KHz and from 65 KHz to 72 KHz. To demonstrate the performance of the Easy Model for loaded PZT, a representative frequency range from 12 KHz to 20 KHz is selected for experiments.

The component values for loaded PZT are obtained in the same manner for unloaded PZT covered in Section IV.A. There are 50 resonant frequencies in the selected frequency range. The dotted lines in Figure 14 show the estimated impedance of loaded PZT based on the Easy Model. From a visual inspection, the Easy Model follows the trend of the measured impedance of loaded PZT in all the four values, resistance, reactance, magnitude, and phase.



(a) Resistance and Reactance



(b) Magnitude and Phase

Figure 14 Loaded PZT impedance – Measurement vs. Easy Model

For a quantitative comparison, correlation coefficients between the measured impedance and the modeled impedance are calculated separately for resistance, reactance, magnitude, and phase, as summarized in Table 3. The Easy Model estimates the loaded PZT impedance with an average agreement rate of 93%. A higher agreement rate is expected if the frequency resolution of the measured impedance is increased.

TABLE 3 LOADED PZT CORRELATION COEFFICIENTS – MEASUREMENT VS. EASY MODEL

Impedance Component	Correlation Coefficient
Resistance	0.9743
Reactance	0.8888
Magnitude	0.9269
Phase	0.9409

C. *Revisit to Unloaded Model*

Based on the Easy Model accommodating multiple resonant frequencies, the unloaded PZT estimation conducted in Section IV.C is revisited to incorporate a local resonant frequency using the model developed for loaded PZT illustrated in Figure 13. We considered two RLC tank circuits accounting for the primary and local resonance frequencies. The values for two tank circuits based on measurements are given in Table 4. Note that the values for a primary parallel resonance remain the same.

TABLE 4 THE EASY MODEL CIRCUIT COMPONENTS – UNLOADED PZT WITH TWO RESONANT FREQUENCIES

R_0 (Ω)	5	R_1 (Ω)	115	R_2 (Ω)	15
C_0 (μF)	0.15	C_1 (μF)	0.277	C_2 (μF)	1.727
		L_1 (μH)	30.253	L_2 (μH)	3.211

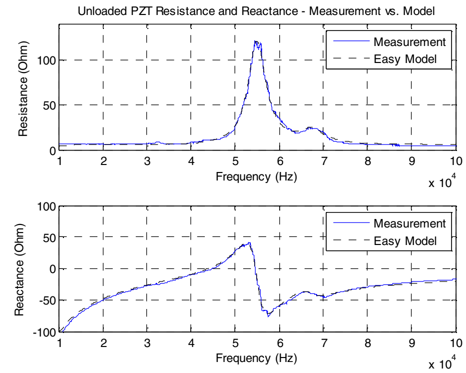
The revised model for unloaded PZT with two resonant frequencies offers a better agreement with the measured impedance as shown in Figure 15. The correlation between the estimated impedance and the measured one is given in Table 5. The average correlation improves to nearly 100 % compared to 99 % for the previous model with only a primary resonant circuit.

TABLE 5 UNLOADED PZT CORRELATION COEFFICIENTS WITH TWO RESONANT FREQUENCIES – MEASUREMENT VS. EASY MODEL

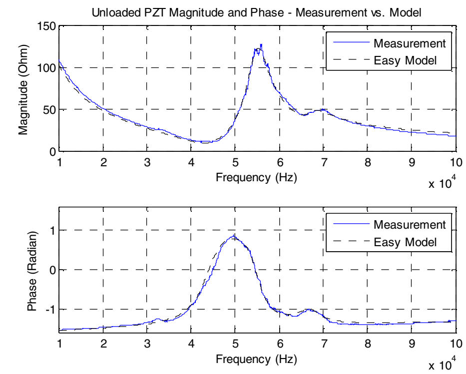
Impedance Component	Correlation Coefficient
Resistance	0.9969
Reactance	0.9963
Magnitude	0.9958
Phase	0.9966

VI. CONCLUSION

This paper proposes an equivalent circuit model that can represent the electrical behaviors of unloaded and loaded piezoelectric ceramics. A circuit model offers easy analysis and quick performance estimation of sensory systems without lengthy measurements. The proposed model can be developed solely from the measured impedance of PZT and does not require any information about the material itself. The proposed model also provides high accuracy, average 99 % or above agreement rate for an unloaded PZT ceramic and 93 % for a loaded piezoelectric ceramic. The modeling procedure is straightforward for easy automation.



(a) Resistance and Reactance



(b) Magnitude and Phase

Figure 15 Unloaded PZT impedance with two resonant frequencies – Measurement vs. Easy Model

ACKNOWLEDGMENT

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